Online supplement to Baillon, Bleichrodt, & Cillo (2015). "A Tailor-Made Test of Intransitive Choice." Operations Research 63(1), 198-211

This online supplement presents additional results and details. In Section 1, we show that our results are inconsistent with González-Vallejo's (2002) proportional difference model. In Section 2, we explain the bisection procedure that we used to measure indifferences in the experiment.

1. The proportional difference model

González-Vallejo's (2002) proportional difference (PD) model is also based on the notion of similarity, but it embeds a deterministic similarity core in a stochastic framework. Consider two acts X = $(p, x_1; 1-p, x_2)$ and Y = $(p, y_1; 1-p, y_2)$ with $x_1 > y_1$, $x_2 < y_2$, $p_1 < ½$. According to the PD model of González-Vallejo (2002, Eq. (3) and the extension to more than two attributes discussed on page 140), X is strictly preferred to Y if and only if

$$\frac{x_1 - y_1}{x_1} - \frac{y_2 - x_2}{y_2} - \frac{1 - 2p_1}{1 - p_1} \geq \tau + \varepsilon. \qquad (1)$$

In Eq.(5), τ is the decision maker's decision threshold. González-Vallejo (2002) suggests that τ can depend on the context and on the decision task. However, within tasks τ is constant. The parameter ε is a random noise term with mean zero. Equation (5) says that X will be preferred to Y if the difference between the proportional advantage of X over Y ($\frac{x_1 - y_1}{x_1}$) and the proportional advantage of Y over X ($\frac{y_2 - x_2}{y_2} + \frac{1 - 2p_1}{1 - p_1}$) exceeds the decision threshold plus error.

The first part of our measurement procedure permits a test of the PD model. Under the PD model the indifferences between $(p,x_{j+1}; 1-p,r)$ and $(p,x_j; 1-p,R)$ imply that:

$$\left|\frac{x_{j+1} - x_j}{x_{j+1}} - \frac{R - r}{R} - \frac{1 - p - p}{1 - p}\right| \leq \tau + \varepsilon. \quad (2)$$

Eq. (2) implies that $\frac{x_{j+1} - x_j}{x_{j+1}}$ should be constant up to random noise for successive elements of the standard sequence.

The data are inconsistent with this prediction. Table 1 shows that the ratio $\frac{x_{j+1} - x_j}{x_{j+1}}$ decreases over the standard sequence. The null hypothesis that $\frac{x_{j+1} - x_j}{x_{j+1}}$ is constant up to random noise for different j could clearly be rejected (repeated measures ANOVA, $p < 0.01$).

Table 1: Mean values of the elicited standard sequence.

x_0	x_1	x_2	x_3	x_4	x_5
20	**32.55** [28.38,33.50]	**45.05** [36.38,48.50]	**60.39** [46.62,60.75]	**74.95** [55.88,75.50]	**89.57** [66.50,85.50]
$\frac{x_{j+1} - x_j}{x_{j+1}}$	**0.357** [0.295,0.403]	**0.251** [0.191,0.295]	**0.222** [0.183,0.255]	**0.174** [0.137,0.185]	**0.153** [0.132,0.171]

Note: **interquartile ranges in square brackets.**

2. Procedure to measure the indifference values

To elicit the standard sequence of outcomes in the first part of our procedure, outcomes x_{j+1} were elicited such that $(⅓, x_{j+1}; ⅔, 11) \sim (⅓, x_j; ⅔, 16)$.[i] The indifference value x_{j+1} was determined through a series of choices between A = (⅓, t; ⅔, 11) and B = $(⅓, x_j; ⅔, 16)$ where t was always an integer and varied as follows. The initial value of t was a random integer in the interval $[x_j, x_j + 25]$. There were two possible scenarios:

(i) If A was chosen we increased t by €25 until B was chosen. We then halved the step size and decreased t by €13. If A [B] was subsequently chosen we once again halved the step size and increased [decreased] t by €6, etc.

(ii) If B was chosen we decreased t by $D' = (t - x_j)/2$ until A was chosen. We then increased t by $D'/2$. If A was subsequently chosen then we increased [decreased] t by $D'/4$, etc.

The elicitation ended when the difference between the lowest value of t for which B was chosen and the highest value of t for which A was chosen was less than or equal to €2. The recorded indifference value was the midpoint between these two values. Table 2 gives an example of the procedure for the elicitation of x_1 through choices between A = (⅓, t; ⅔,11) and B = (⅓,20; ⅔,16). In this example, the initial random value for t was 36. The recorded indifference value was the midpoint of 26 and 28, that is, 27.

Table 2. Example of the elicitation of x_1.

Iteration	t	Choice
1	36	A
2	28	A
3	24	B
4	26	B

The procedure in the second part was largely similar. We elicited the value z_p for which indifference held between A = $(p,x_4; 1-p,20)$ and B = $(p,x_3; 1-p, z_p)$[ii] where p was one of {¼, ⅖, ⅗, ¾} and x_4 and x_3 were the outcomes of the standard sequence elicited in the first part. The indifference value was elicited through a series of choices between A = $(p,x_4; 1-p,20)$ and B = $(p,x_3; 1-p, s)$, where s was always an integer and never equal to x_3 to avoid the possibility of event-splitting effects. The initial stimulus s was a random integer in the range $[z_{EV}-3, z_{EV}+3]$ where z_{EV} is the value of s that makes A and B equal in expected value with the restriction that s could not be less than €20. There were two possible scenarios:

(i) As long as A was chosen we increased s by $D = (x_4 - z_{EV})/2$ if $p \leq ½$ and by $D = (x_5 - z_{EV})/2$ if $p > 1/2$. We used a different adjustment for $p \leq ½$ to avoid violations of stochastic dominance. We kept increasing s by this amount until B was chosen. Then we decreased s by D/2. If A [B] was subsequently chosen we increased [decreased] s by D/4, etc. A special case occurred if the difference between s and x_4 (for $p \leq ½$) or between s and x_5 (for $p > ½$) was less than 5. Then we increased s by 10 and subsequently kept increasing s by 5 until B was chosen. Then we decreased s by 3.

(ii) If B was chosen we decreased s by $D' = (s - 20)/2$ until A was chosen. We then increased s by D′/2. If A [B] was subsequently chosen we increased [decreased] s by D′/4, etc.

Table 3. Example of the elicitation of $z_{¼}$ when $x_4 = 61$ and $x_3 = 48$.

Iteration	s	Choice
1	26	A
2	44	B
3	35	B
4	31	B
5	29	A

The remainder of the procedure was the same as in the elicitation of u. The elicitation ended when the difference between the lowest value of s for which B was chosen and the highest value of s for which A was chosen was less than or equal to €2. The recorded indifference value was the midpoint between these two values. Table 3 gives an example of the procedure for the elicitation of $z_{¼}$. In the example, the initial choice was between A = (¼, 61; ¾, 20) and B = (¼, 48; ¾, 26), where 26 was selected as the initial stimulus value from the interval [24.3 −3, 24.3 +3]. The recorded indifference value was 30, the midpoint between 29 and 31.

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[i] In the experiment we varied what was option A and what was option B.

[ii] In the experiment we varied which option was A and which B.